# Assertional Logic

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# Content

- Motivation
- Syntax and semantics
- Extensions by definition
- Incorporating FOL, probability and time
- AL vs FOL
- Potential applications
- Conclusion

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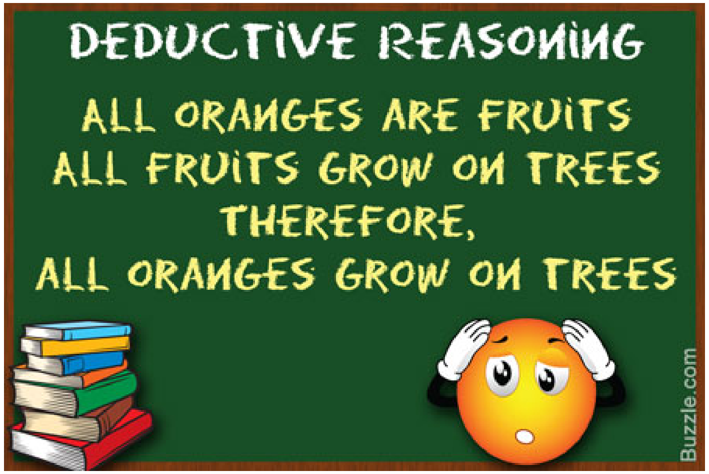
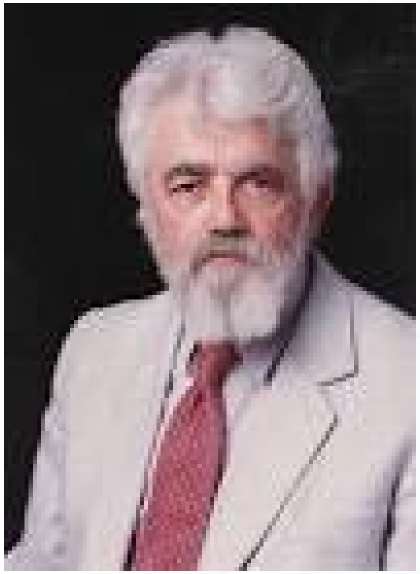
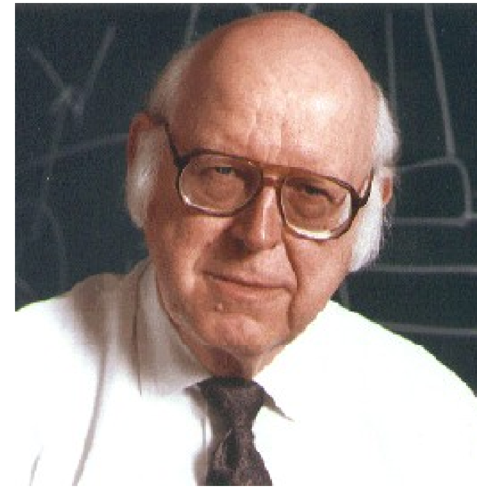
# Symbolic AI

| P | Q | P∧Q | P∨Q | P→Q |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | F | T | T |
| F | F | F | F | T |



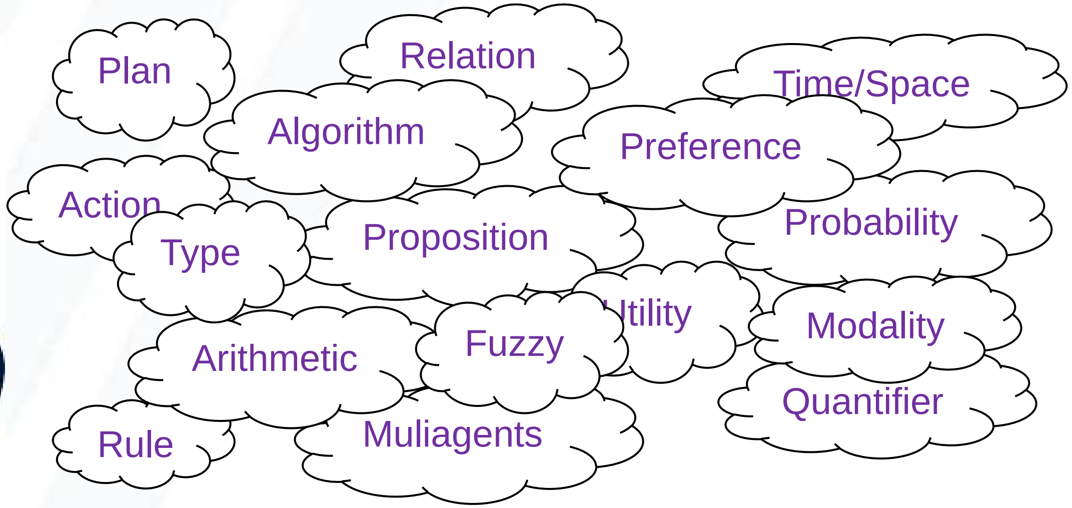




# But ...


# Representation




Problem: one more building block, much more effort
Challenge: how to make them living happily ever after

# Reasoning

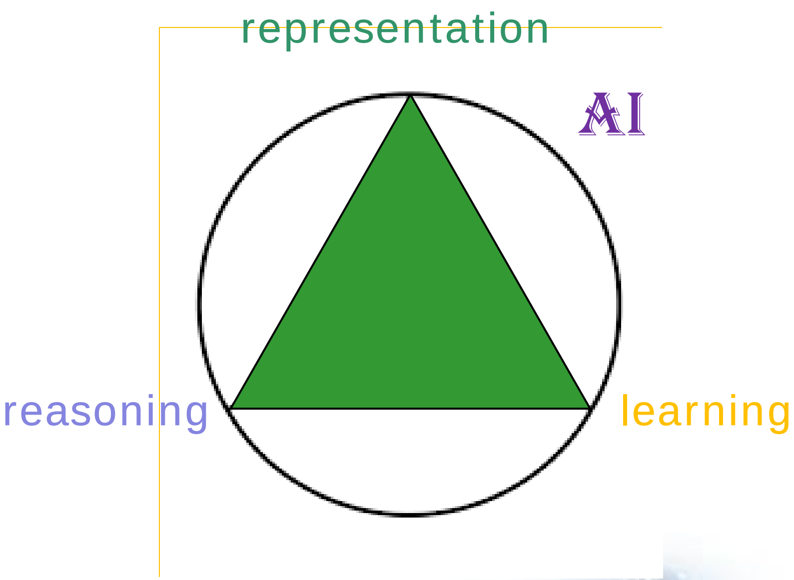

Problem: more expressive less efficient, more efficient less expressive
Challenge: both are needed but there is no free lunch

# Learning


Problem: KR reasoners are algorithm based, little power to learn
Challenge: learnable reasoning

# 6E: What we need

Elegant
Extensible
Expressive
Efficient
Educable
Evolvable



# Representation – a case study

Situation calculus (FOL + action)

- ✓ ∀a,v,w,s: affects(a, on(v,w), s) → ∃x,y: a=move(v,x,y)
- ✓ ∀a,v,s: affects(a, clear(v), s) → (∃x,z: a=move(x,v,z)) ∨ (∃x,y: a=move(x,y,v))
- ✓ ∀a,v,w,s: affects(a, colour(v,w), s) → ∃x: a=paint(v,x)

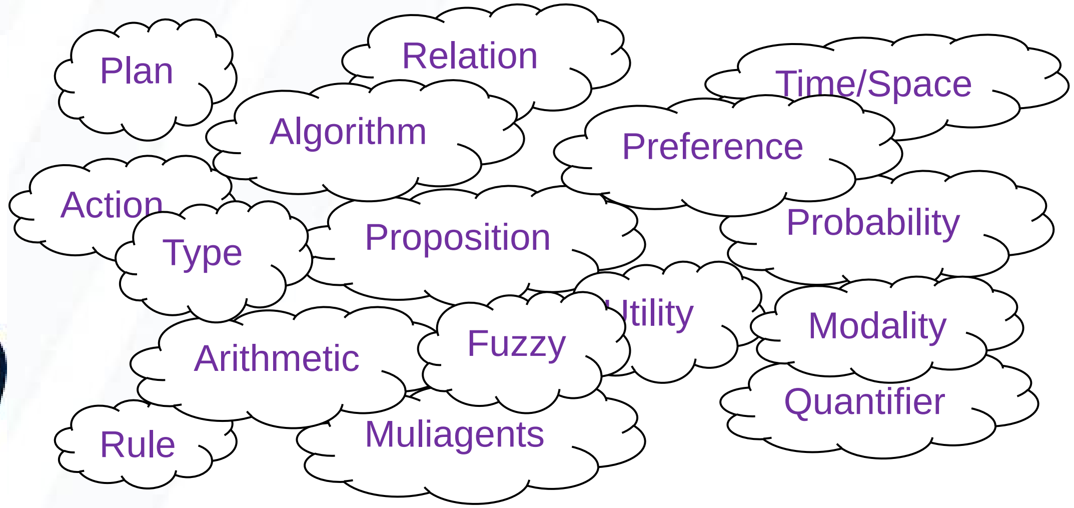
Advantages

- ➢ clear, no ambiguity
- ➢ doable, e.g., Golog

Issues

- ➢ needs to define a completely new syntax and semantics
- ➢ too complicate - no outsider can understand it
- ➢ still has problems, e.g., the frame problem, the ramification problem
- ➢ not that expressive – cannot quantifier over predicates, formulas
- ➢ not efficient at all
- ➢ too many more building blocks to be incorporated,
  e.g., plan, probability, time.

# Representation




Problem: one more building block, much more effort

Challenge: how to make them living happily ever after

Solution: assertional logic

Elegant
Extensible
Expressive

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# Assertional Logic

| | | Syntax | | Semantics |
|---|---|---|---|---|
| | | Individual | | Element |
| Formalization | → | Concept | → | Set |
| | | Operator | | Function |
| Knowledge | → | a=b | → | I(a)=I(b) |
| | | x or $O(x_1,\ldots,x_n)$ | | |

2+3=6

Father(Alice)=Bob

Alice∈Human=Ture

Male∩Human⊆Animal=False

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# Definition - Individual

i=a

1=Succ(0)

# Definition - Operator

$$Op(C_1,\ldots,C_n)=T(C_1,\ldots,C_n)$$

$$Succ(n)=\{n,\{n\}\}$$

$$Uncle(x)=Brother(Father(x))$$

# Definition - Concept

Enumeration

C={i1,…,in}

Digit={1,2…,9}

Operation

C=C1∩C2

Man=Human∩Male

Comprehension

C'=C|A(C)

Male=Animal|Sex(Animal)=Male

Replacement

C'=Op(C)

Parent=ParentOf(Human)

# Multi-Assertions

$M_n(a_1=b_1,\ldots,a_n=b_n)::=(a_1,\ldots,an)=(b1,\ldots,bn)$

$$\text{M-A}::= \bigcup_{1\leq i\leq\infty} M_i(\mathcal{A}^1,\ldots,\mathcal{A}^i)$$

# Nested Assertions

$$Nested - Term ::= Term \cup Op(Nested - Term)$$

$$Nested{-}Assertion ::= Nested{-}Term = Nested{-}Term.$$

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# Propositional Connectives

$$\neg(a = a') ::= \{a\} \cap \{a'\} = \emptyset$$
$$\wedge(a = a', b = b') ::= (\{a\} \cap \{a'\}) \cup (\{b\} \cap \{b'\}) = \{a, a', b, b'\}$$
$$\vee(a = a', b = b') ::= (\{a\} \cap \{a'\}) \cup (\{b\} \cap \{b'\}) \neq \emptyset$$
$$\rightarrow (a = a', b = b') ::= (\{a, a'\} \setminus \{a\} \cap \{a'\}) \cup (\{b\} \cap \{b'\}) \neq \emptyset$$
$$\equiv (a = a', b = b') ::= \wedge(\rightarrow (a = a', b = b'), \rightarrow (b = b', a = a')).$$

# Quantifiers

$$
\begin{aligned}
\forall(C, A(C)) \quad &::= \quad C|A(C) = C \\
\exists(C, A(C)) \quad &::= \quad C|A(C) \neq \emptyset
\end{aligned}
$$

# (Conditional) Probability

$$Pr(A) = \frac{\Sigma_{w,w \models A} \; W_w}{\Sigma_w \; W_w}.$$

$$Pr(A_1|A_2) = \frac{\Sigma_{w,w \models A_1, w \models A_2} \; W_w}{\Sigma_{w,w \models A_2} \; W_w}.$$

# Time (Point)

$$\mathcal{T}(a = b, tp) ::= \mathtt{t}(a, tp) = \mathtt{t}(b, tp).$$

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# AL vs High-Order Logic

$$\forall(C, A(C)) \quad ::= \quad C|A(C) = C$$
$$\exists(C, A(C)) \quad ::= \quad C|A(C) \neq \emptyset$$

| **C Concept of** | **HOL** |
|---|---|
| Same individuals | FOL |
| Different individuals | Many sorted FOL |
| Concepts | Monadic SOL |
| Operators | SOL |
| Operators of … operators | HOL |
| Assertions | ??? |

Elegant
Expressive
Extensible

# AL for knowledge representation

Advantages

- clear, no ambiguity ✓
- doable ✓

Issues

- needs to define a completely new syntax and semantics ✖
- too complicated - no outsider can understand it ✖
- not that expressive ✖
- too many more building blocks to be incorporated extensible
- not efficient at all TBA

# AL vs Description Logic I

| Constructs | Description logic | Our approach |
|---|---|---|
| individual | individual | individual |
| concept | concept | concept |
| role | Role | binary Boolean operator |
| intersection | $C \sqcap D$ | $C \cap D$ |
| union | $C \sqcup D$ | $C \cup D$ |
| complement | $\neg C$ | $\mathcal{I} \setminus C$ |
| reverse role | $R^-$ | $R^-(C, D) ::= R(D, C)$ |
| existential restriction | $\exists R.C$ | $\mathcal{I} \| \widehat{R^-}(\mathcal{I}) \cap C \neq \emptyset$ |
| universal restriction | $\forall R.C$ | $\mathcal{I} \| \widehat{R^-}(\mathcal{I}) \subseteq C$ |
| at least restriction | $\geq nR.C$ | $\mathcal{I} \| (\widehat{R^-}(\mathcal{I}) \cap C)^C \geq n$ |
| nominal | $\{a\}$ | $\{a\}$ |
| concept assertion | $C(a)$ | $a \in C$ |
| role assertion | $R(a, b)$ | $R(a, b)$ |
| individual equality | $a \approx b$ | $a = b$ |
| concept inclusion | $C \sqsubseteq D$ | $C \subseteq D$ |

# AL vs Description Logic II

| AL | Description Logic |
| --- | --- |
| n-ary operators | Binary relations |
| Comprehension/Replacement | Role Restriction |
| Complex assertions/quantifiers | Fragment of FOL |
| a=b | Different kinds |
| Extensibility by definition | Extensibility by new components |

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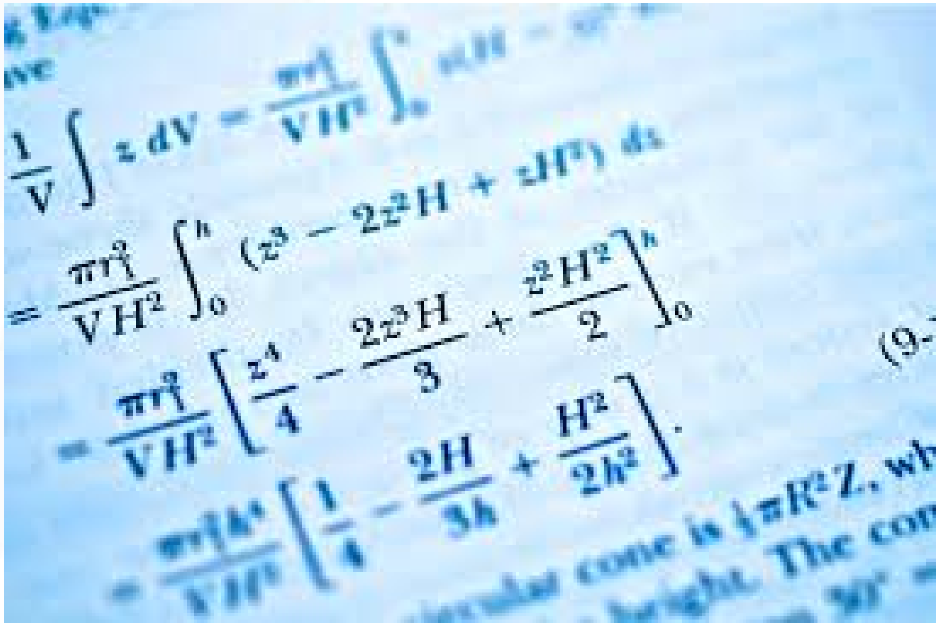
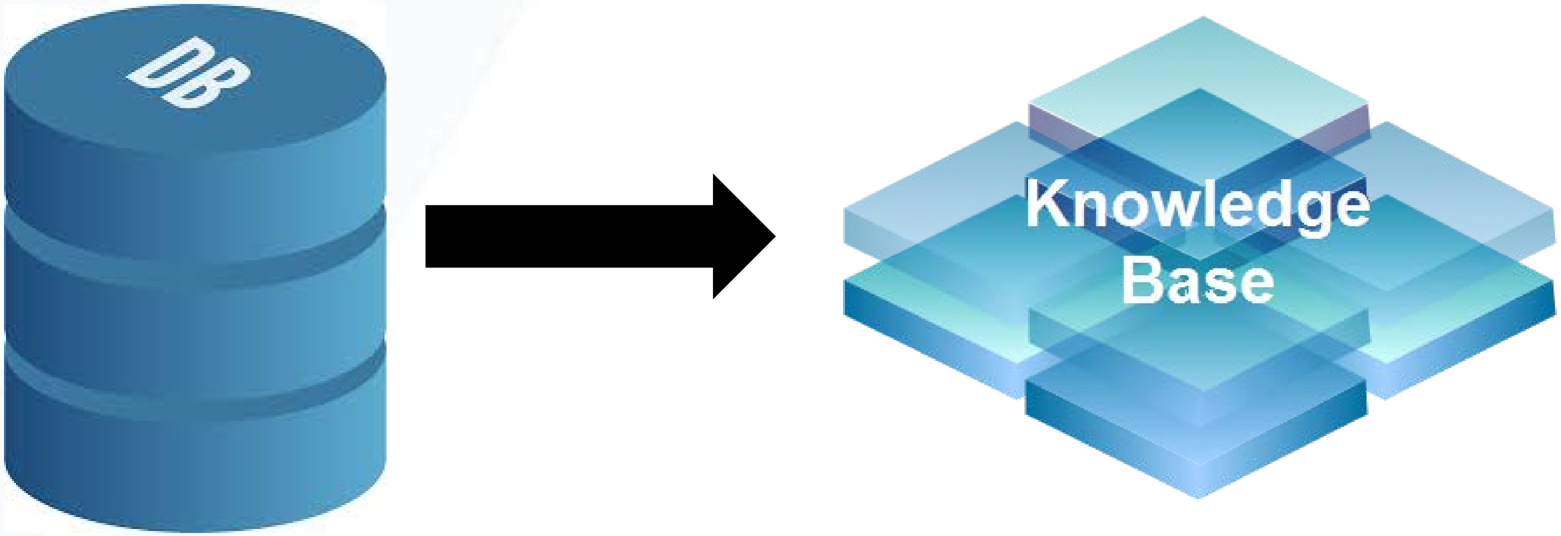
# Mathematics test



# Natural language formalization


Noun: Concept
Adjective: unary Boolean operator

# From database to knowledge base




$$Op(a_1 \ldots a_n) = b$$

$$a \in C$$

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# Concluding Remarks

- Knowledge representation: from FOL to AL
- Why FOL is not perfect?
- Elegancy, Extensibility and Expressivity
- AL – syntax and semantics
- AL – extensibility via definitions
- Why AL is better than FOL?

# More on AL representation

- actions and their effects
- plans, method and hints
- incomplete information and null

# Reasoning


Problem: more expressive less efficient, more efficient less expressive
Challenge: both are needed but there is no free lunch

Solution: reasoning by knowledge

Efficient

# Learning


Problem: KR reasoners are algorithm based, little power to learn
Challenge: learnable reasoning
Solution: learnable knowledge

**E**ducable
**E**volvable

*Thank you!*